

Convergences characterizing the (semi)continuity of the limits of functions

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Abstract

One of the central question of Analysis was: what precisely must be added to pointwise convergence of a sequence of continuous functions to preserve continuity of the limit? In 1841 Weierstrass discovered that uniform convergence yields continuity of the limit function. In 1883 Arzelà found a necessary and sufficient condition under which the pointwise limit of a sequence of real valued continuous functions on a compact interval is continuous. His work initiated a study that led to several outstanding papers. Recently, in 2009, in the realm of metric spaces, Beer and Levi found another necessary and sufficient condition through the novel notion of strong uniform convergence on finite sets. We survey a wide range of deep generalizations and focus on recent directions of investigation in this area, namely the statistical approach and new convergences characterizing the semicontinuity of the limit function.