Jointly continuous utility functions on k_ω-spaces

Rita CEPPITELLI

Let (X, τ) be a submetrizable k_ω-space, that is the inclusion inductive limit of a decreasing sequence $(F_n)_n$ of second countable and locally compact subspaces of X. Of course the family $(F_n)_n$ determines the topology of X. It is well known that X is a quotient space of a locally compact second countable space.

These spaces seem to be very interesting in the study of the utility representation problem. In [3] the authors prove the existence of a continuous representation of a topological space that is inclusion inductive limit of a countable chain of compact subspaces $(X_n)_n$ and $\preceq$ is a preorder on X such that every $\preceq|_{X_n}$ is closed and order-separable. In [4, 2] it is proved that each closed preorder on X has a continuous utility representation and some well known theorems due to Levin [7] on the existence of the jointly continuous utility functions are generalized on submetrizable k_ω-spaces.
Back in [1], using a result of Levin, proves the existence of a continuous map from the space of preorders topologized by closed convergence and the space of utility functions with different choice sets (partial maps) endowed with a generalization of the compact-open topology. The commodity space is locally compact and second countable. Recently Back's Theorem is generalized to non-metrizable commodity spaces, precisely, to a regular space submetrizable by a boundedly compact metric in [6] or to X submetrizable k_ω-space in [5].

The continuous utility representation theorems on submetrizable k_ω-spaces can have some economic implications, in fact an example of submetrizable k_ω-space is the space of tempered distributions, which seems to be of interest in the study of market models in the Decision Theory.

References

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Rita Ceppitelli
Department of Mathematics, University of Perugia
Via L. Vanvitelli, 1 - 06123 Perugia - Italy
Tel: +39-0755855032
Fax: +39-0755855024
e-mail: matagria@dmi.unipg.it