

# Hausdorff excess and point-free geometry

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## Abstract

In the Euclidean geometry points are the primitive entities. Point-based spatial construction is dominant but apparently, in a constructive point of view and a naïve knowledge of space, the region-based spatial theory is more quoted, as recent and past literature strongly suggest. The point-free geometry refers directly to sets, the *spatial regions*, and *relations between regions* rather than referring to points and sets of points. One of the approach to point-free geometry proposes as primitives the concept of region and quasi-metric, a non-symmetric distance between regions, yielding a natural notion of diameter of a region that, under suitable conditions, allows to reconstruct the canonical model. The intended canonical model is the hyperspace of the non-empty regularly closed subsets of a metric space equipped with the Hausdorff excess. The canonical model can be enriched by adding more qualitative structure involving a distinguished countable subfamily of *bounded regions*, and a group of *similitudes* preserving bounded regions, so eventually producing a metric geometry whose points, roughly speaking decreasing sequences of bounded regions with vanishing diameters, have some specific features preserved by similitudes and different metric geometries for distinct bounded regions.

**A. Di Concilio and G.G. Gerla** *Quasi-metric spaces and point-free geometry*  
Math. Struct. in Comp. Science (2006) vol.16 pp.115-137.