A genesis of interval orders and semiorders: transitive NaP-preorders and NaP-preferences

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Abstract

A *NaP-preorder* (necessary-and-possible preorder) on a set A is a pair $\left(\succsim^N, \succsim^P\right)$ of binary relations on A, whose necessary component $\succsim^N$ is a partial preorder and whose possible component $\succsim^P$ is an extension of $\succsim^N$ satisfying a mixed form of completeness. We study several forms of mixed transitivity of NaP-preorders, i.e., finite sequences of $\succsim^N$ and $\succsim^P$ that imply $\succsim^P$.

A *NaP-preference* is a NaP-preorder on A satisfying a mixed transitivity property for sequences of length two. NaP-preferences on A can be characterized in a natural way: in fact, a pair $\left(\succsim^N, \succsim^P\right)$ of binary relations on A is a NaP-preference if and only if there exists a nonempty family $\mathcal{R}$ of total preorders on A such that $\succsim^N = \bigcap \mathcal{R}$ and $\succsim^P = \bigcup \mathcal{R}$.

We connect transitive types of NaP-preorders and NaP-preferences to binary relations that are well-known in preference theory, namely, *interval orders* and *semiorders*. In fact, we show that interval orders and semiorders are the possible components of NaP-preorders and NaP-preferences satisfying suitable mixed transitivity properties of length three. We also introduce strong versions of interval orders and semiorders, characterized by forms of mixed transitivity of length four.

Further, we give a *geometric visualization* of all these types of preference relations in some special cases. Finally, we hint some possible applications of suitably transitive NaP-preferences in the case that the space of partial preorders on a set is endowed with a metrizable topology.