TREES AND SOME CLASSES OF CORSON COMPACTS

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Recall that the class of Gul'ko compact spaces consists of compacts X admitting a special embedding into a Σ-product of the real lines $\Sigma(\mathbb{R}, T)$: there exists a partition of a set $T = \cup_{n=1}^{\infty} T_n$ such that $\cup\{T_n : |supp(x) \cap T_n| < \aleph_0\} = T$ for any $x \in X$, where $supp(x) = \{t \in T : x_t \neq 0\}$. Class of Gul'ko compacts is a proper subclass of the class of Corson compacts.

We define the following

Definition 1. *A compact space X is called* **almost** *Gul'ko compact if X embeds into $\Sigma(\mathbb{R}, T)$ so that $\cup\{T_n : |supp(x) \cap T_n| < \aleph_0\} = T \setminus D_x$, where D_x is at most countable set for any $x \in X$.*

Consider the family $\mathcal{A}$ of all chains of a partially ordered set $(T, <)$ and define the following compact subspace of the Cantor cube $X_T = \{\chi_B : B \in \mathcal{A}\} \subset \{0,1\}^T$, where $\chi_B(t) = 1$ iff $t \in B$.

Clearly, X_T is a Corson compact iff T has no uncountable chains. Let T be a tree, then X_T is a Gul'ko compact iff T is special tree, i.e. T is a countable union of antichains.

We investigate topological properties of almost Gul'ko compact spaces and explore trees T such that the space X_T is an almost Gul'ko compact.

Definition 2. *We say that a tree T is A-special if there exists a partition $T = \cup_{n=1}^{\infty} T_n$ for which the set*

$$T \setminus \cup\{T_n : |A \cap T_n| < \aleph_0\}$$

is at most countable for any chain $A \subset T$.

Theorem 1. *Each normal A-special tree contains a dense subset which is a countable union of antichains.*

Example 1. Let S be a bistationary subset of ω_1. By $T = T(S)$ denote the tree of all countable closed in ω_1 subsets of S. Then T is not A-special.

Example 2. Let E be a linearly ordered set. By σE we denote the set of all bounded well ordered subsets of E ordered as follows: $s < t$ iff s is a proper initial segment of t. If Q denotes the rationals, then σQ is an A-special tree.

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