Total boundedness in metrizable spaces
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We show that a metric space (X, d) is separable if and only if

the bornology of its $d-$bounded subsets agrees with the bornology of $\rho-$totally bounded subsets with respect to some equivalent metric ρ. We also show that the bornology of $d-$totally bounded subsets agrees with the bornology of $\rho-$bounded subsets with respect to some equivalent metric if and only if the former bornology has a countable cofinal family. Finally, we characterize those bornologies on a metrizable space that are bornologies of totally bounded sets for some metric compatible with the topology.