

## NEW TYPES OF COMPLETENESS

**M. Isabel Garrido and Ana S. Meroño**

Universidad Complutense de Madrid

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Cauchy sequences and Cauchy nets characterize total boundedness and completeness in metric and uniform spaces, respectively. If residuality of the indexes in the definition of Cauchy sequence or net is changed by cofinality, one has the notion of cofinally Cauchy for sequences and nets. Thus, by asking the clustering of them one gets a stronger type of completeness, known as cofinal completeness. Natural problems related to the above notions are the uniformization and the metrization of a Tychonoff space by a complete or cofinally complete uniformity and metric (see Beer [1] and Howes [5]).

Similarly, we start here from a uniform notion of boundedness in uniform and metric spaces called Bourbaki-boundedness (see, for instance, Hejzman [3]) which is weaker than total boundedness. We give a characterization of this notion in terms of what we call Bourbaki-Cauchy sequences and nets. Therefore, by asking the clustering of them, we will have also a stronger type of completeness, called *Bourbaki-completeness*. Again, by changing residuality by cofinality, we obtain the corresponding notion of *cofinally Bourbaki-completeness*.

Furthermore, when studying the problem of uniformization and metrization in this setting, we find the relation of Bourbaki-completeness with the concept of  $\delta$ -completeness introduced by García-Máynez in [2], and the relation of cofinal Bourbaki-completeness with strong-paracompactness studied by Hohti in [4].

### REFERENCES

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