

# Some Characterizations of $C(K)$ as Ordered Normed Algebra

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## Abstract

It is well known that if  $A$  is a normed algebra, and also a uniformly closed  $\Phi$ -algebra, then  $A$  is a strictly real algebra and therefore its topological spectrum  $K$  is a nonempty and compact set [3],[2]. In this talk it is shown that if  $A$  is as above and satisfies the condition: “Every closed order interval is a bounded set”, then  $A$  is isomorphic and homeomorphic to algebra  $C(K)$  with respect to the norm of the uniform convergence. In particular  $A$  is a Banach algebra. In addition, using some results due to F. Albiac and N. Kalton [1], a reciprocal of the previous result can be obtained: On a real and unitary Banach algebra  $A$  one defines the relationship  $a \leq b \Leftrightarrow b - a \in \text{cl}\{a^2 : a \in A\}$ . Then, if  $A$  is a strictly real algebra, and each interval closed under  $\leq$  is a bounded set, then  $\leq$  is a compatible order on  $A$  and  $A$  is isomorphic and homeomorphic to its spectral representation  $C(K)$ .

## References

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