Quasi-metric structures applied to analysis of complexity

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In 1995, M. Schellekens began the development of a mathematical model to analyze the algorithmic complexity based on the construction of a quasi-metric defined on the space of the complexity [M. Schellekens, The Smyth completion: a common foundation for denotational semantics and complexity analysis, in:Proc. MFPS 11, Electronic Notes in Theoretical Computer Science 1 (1995), 211-231]. This model provide an adequate computational interpretation of the fact that a program or an algorithm is more efficient than other in all of its inputs, however this framework does not give a computational interpretation of the fact that a program or an algorithm is asymptotically more efficient than another. The fuzzy quasi-metric spaces provide a parameter "t" such that a suitable use of this ingredient may give rise to extra information on the involved computational process; thus we introduce the concept of complexity fuzzy quasi-metric space, which provides a successful model to interpret the asymptotic efficiency of the complexity functions. In this contest we present some fixed-point theorems by using appropriate notions of completeness and we apply this approach to deduce the existence of solution for some recurrence equations associated to the analysis of Quicksort algorithms and Divide & Conquer algorithms, respectively. Finally we present other quasi-metric structures that have been successfully used to analyze the complexity of programs and algorithms.